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Mathematical Modelling of Wastewater Treatment: A Review and Application

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

The Sewon domestic wastewater treatment plant (WWTP) plays a crucial role as a domestic wastewater treatment unit in the densely populated areas of Yogyakarta. Domestic wastewater generally contains suspended solids, organic matter, pathogens, and nutrients. Uncontrolled management of domestic wastewater effluent can cause serious environmental impacts, such as the spread of diseases and a decrease in water quality that has the potential to hinder reuse. Sedimentation and eutrophication processes can also occur, further impacting the performance of wastewater treatment units. The phenomena and challenges that occur in WWTPs can be examined through mathematical modeling, especially dynamic models, to understand the relationship between waste components and the working mechanism of the wastewater treatment system. Numerical approaches, especially the finite difference method, play a significant role in

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solving the complexity of models that are difficult to solve using exact methods. This article presents a study based on field observations and analysis of relevant scientific literature, to identify various potentials and challenges in wastewater management, analyzed through mathematical model approaches and numerical techniques.

Keywords: Wastewater treatment; mathematical models; WWTP; domestic wastewater.

1 Introduction

The 6th Sustainable Development Goal (SDG) aims to achieve safe sanitation by 2030. However, adequate household wastewater collection and treatment facilities remain a challenge, especially in Southeast Asian countries (Arcowa, 2018; Bongartz et al., 2016; McConville, 2011; Rosemarin et al., 2016). Water contamination and pollution represent serious global environmental problems, with adverse impacts on human health, food production, economic growth, and the natural environment (Mian et al., 2023; Tauseef et al., 2023).

The concentration of settlements in specific areas creates various new problems, especially in the liquid waste sewer system (Sudharto & Samekto, 2007). The development of cities characterized by economic growth, population increase, and expansion of urbanization in urban areas is the main factor causing environmental pollution (Smol, Adam, et al., 2020; Zhang et al., 2017). The denser an area, the more complex the pollution problems, especially those caused by household wastewater or domestic wastewater from settlements. If this pollution is not addressed properly, the consequences can be severe. Uncontrolled domestic wastewater has polluted almost all rivers in Indonesia, especially in Java (Sudharto & Samekto, 2007; Sunarsih & Sutrisno, 2023).

Water pollution is caused by the improper discharge of domestic and industrial wastewater effluents into the environment, either directly or indirectly, without adequate treatment. Numerous studies have revealed the challenges faced in wastewater treatment (Afolalu et al., 2022). Wastewater treatment is an important step to ensure the water is safe before it is reused in various applications, such as agricultural irrigation, road cleaning, firefighting, geothermal production, industrial processes, commercial washing, and construction (Eslami et al., 2018). One of the main ways to manage domestic wastewater effluent is through the use of centralized wastewater treatment plant (WWTP) units (Sunarsih & Sutrisno, 2023). This paper focuses on identifying key gaps and challenges in wastewater treatment, with an analysis based on mathematical models and approaches that aim to optimize the treatment process while minimizing its impact on the environment.

The article is organized systematically to provide a comprehensive understanding of the topics covered. The introduction outlines the background of the research, including the importance of wastewater treatment in supporting environmental sustainability and its relevance to achieving SDG 6. In addition, the introduction explains the main objective of the research, which is to develop and apply a mathematical model to improve the efficiency of WWTP systems. The structure of the article includes a literature review on effluent profiles and components, the application of mathematical models in WWTPs, numerical approaches to solve the mathematical models, and conclusions that highlight the implications of the research and its potential future applications.

2 Study Result

2.1 Brief profile of Sewon WWTP

The Sewon WWTP, as shown in Fig. 1, is one of the centralized domestic wastewater treatment units located in Bantul Regency. The Sewon WWTP service area covers the Kawasan Perkotaan Yogyakarta (KPY), also known as Kartamantul. This area includes almost all of Yogyakarta City, parts of Sleman Regency (Mlati, Depok, Gamping, and Ngaglik sub-districts), and Bantul Regency (Kasihan, Sewon, and Banguntapan sub-districts). The Sewon WWTP has a service capacity of up to 75,000 house connections, with the ability to treat up to 52,000 m^3 per day or about 1,800 m^3 per hour. Domestic wastewater from the region is gravity flowed through a network of pipes, starting from the service pipe to the lateral pipe to the main pipe, with a total network length of 323.29 km. The sewage treatment process at the Sewon WWTP involves several main stages. After the wastewater enters through the inlet, it goes through the stages of equalization, Sequencing Batch Reactor (SBR), maturation, chlorination, and sludge treatment, before finally being discharged through the outlet (Dinas PUP-ESDM, 2022).

Fig. 1. Sewon WWTP in October 2023

Along with the development of Yogyakarta City, several sectors have experienced increased activity, leading to a rise in the volume of liquid waste produced. One example is the home industry sector, including Micro, Small, and Medium Enterprises specializing in Yogyakarta's traditional batik production, which are distributed across various locations in Bantul Regency.

2.2 Characteristics of domestic liquid waste

Liquid waste is the residual material derived from industrial and domestic processes that use water as raw material. The characteristics of liquid waste are determined by its physical, chemical, and biological properties (Britton, 1994). Liquid waste is divided into two main categories: (1) Industrial wastewater, which is liquid waste generated from industrial production processes, (2) Domestic wastewater, which is liquid waste originating from residences, hotels, restaurants, markets, cinemas, retail companies, hospitals, and other places of worship, not from industrial activities (Irianto, 2016). Domestic waste consists of feces from bathrooms, toilets, and kitchens. This waste is a complex mixture of mineral and organic substances in various forms, including large and small particles, solids, residual floating materials, and colloidal and semi-colloidal materials (Kamilah et al., 2022; Martopo, 1987).

The main components of domestic wastewater include suspended solids, biodegradable organic matter, pathogens, and nutrients (Sudharto & Samekto, 2007). A prominent characteristic of urban wastewater is the high organic content, which includes heavy metals (Zheng et al., 2021), nitrogen compounds, phosphorus (Li et al., 2012; Smol, Preisner, et al., 2020), bacteria, algae, zooplankton, detritus organic matter, NH_3, Dissolved Oxygen (DO), total coliforms, fecal coliforms and Biochemical Oxygen Demand (BOD) (P. Sunarsih et al., 2015). The disposal of contaminated domestic and industrial wastewater without proper management can cause various waterborne diseases, such as infectious hepatitis, cholera, and typhoid (Pan et al., 2019).

One of the hazardous components in wastewater is heavy metals. Heavy metals are persistent pollutants with toxic properties that are not biodegradable, so they can accumulate in living organisms (Hafidi et al., 2023). The presence of heavy metals poses a significant risk to water quality and human health, with the potential to cause serious illness (Hafidi et al., 2023; Lü et al., 2019). The main source of heavy metal contamination is the direct discharge of effluents into waterway systems or rivers, which increases the risk of toxicity. Research shows that heavy metals are mostly present in suspended particles in wastewater and surface water (Xu et al., 2022).

Sedimentation can occur in WWTPs. Sedimentation is one of the physical wastewater treatment processes that uses the force of gravity to separate suspended solid particles that have formed from the water (Kristijarti et al., 2013). In wastewater treatment ponds, sediment or sludge deposits can be used to reduce the concentration of suspended particles (Rosemarin et al., 2016). However, excessive sedimentation can cause suboptimal inflow and outflow in the pond, reducing effluent treatment performance.

Eutrophication can also occur in WWTPs. Eutrophication begins with the excessive entry of nutrient compounds, such as nitrogen and phosphorus, resulting from industrial, agricultural, domestic activities, surface runoff, and other sources. These nutrients enter wastewater treatment systems, causing phytoplankton blooms (Ansari et al., 2011; González & Roldán, 2019; Smith & Schindler, 2009).

2.3 Liquid waste volume growth in a mathematical model approach

Interactions between components in liquid waste ecosystems are often studied through a mathematical model approach. One such approach is the optimization model, which is used to optimize the pollutant reduction process in facultative stabilization ponds (S. Sunarsih et al., 2019). The model is designed to determine the optimal volume of treated wastewater and duration of wastewater storage so that the pond efficiency approaches the reference value. Optimization of variables affecting phosphorus content in stabilization ponds has also been conducted (Sells et al., 2018). In addition, a multi-objective approach can represent the reduction of BOD effluent load in WWTPs (Kamilah et al., 2022). This model has three objective functions, namely maximizing the treated BOD effluent load, minimizing the difference between the BOD reduction efficiency value and the reference value, and minimizing aerator power consumption.

In addition to optimization models, approaches with dynamic models have also been developed. For example, a simple mathematical model based on a system of non-linear differential equations describing biological wastewater treatment, taking into account the inflow and outflow during the treatment process (Islami et al., 2022). This model was built using the basic Monod equation. Another dynamic model formulates chemical and biological processes simultaneously, with the rate of change of substance concentration described through a system of non-linear differential equations (P. Sunarsih et al., 2015). This model utilizes the Monod equation with a time correction factor at the maximum growth rate. Dynamic models are considered more effective in representing chemical-biological interactions in wastewater treatment systems. The advantages of this model are the ability to support both analytical and numerical approaches, and predict complex processes with high accuracy.

Another approach is control-based mathematical models, such as a model to minimize wastewater treatment time through aerator control (oxygen transfer coefficient) (Grigor’eva & Khailov, 2020). Another study addressed the control of an aerator machine, which analyzed the limits and continuity of the intervals given in the mathematical model (Bondarenko et al., 2016). Mathematical cell models have also been used to minimize the total electrical energy use in the aeration process at WWTPs (Xu et al., 2022), taking dissolved oxygen and sludge discharge as control variables. These approaches demonstrate the flexibility and potential of mathematical models in improving effluent treatment efficiency, whether through optimization, process dynamics, or operational control.

2.4 Numerical approaches as mathematical model solving

Numerical approaches are often used to solve mathematical models, especially differential equations that are difficult to solve exactly by analytical methods. The difficulty is usually caused by the complexity of the model, for example, if the model contains non-linear terms (Kartono, 2012). Various numerical methods can be used, such as finite difference method, finite element method, finite volume method, Runge-Kutta method, and others. This paper focuses on the finite difference method applied to the WWTP ecosystem. The main advantage of the finite difference method over other methods is its ease of application and its suitability to the relatively simple WWTP geometry, generally in the form of a rectangular pond.

Some studies of finite difference methods applied to WWTP ecosystems include the finite difference method used to simulate sedimentation and flocculation units and predict removal efficiency with the help of Matlab software (Naser & Abdulrazzaq, 2022). The finite difference method can also represent the simulation of the sedimentation process using the Mason-Weaver equation, which can further be used to model the behavior of epithelial cells and the lung system on the chip (Nikolic et al., 2021). The finite difference scheme of this method can be combined with the Lattice Boltzmann Method (LBM) to simulate complex flow through a uniform lattice with low diffusivity so that it can facilitate coupling with finite difference methods (Lemus et al., 2021).

The finite difference method can also be used as a numerical simulation of the Burger equation in two-dimensional nonlinear steady-state mode to model the phenomenon of particle fall velocity in stagnant or nearly stagnant fluids, such as water settling behind a dam that indicates the presence of suspended particles at several positions and times (Rezaei & Rahideh, 2020). Finite difference numerical models can also be modified based on the conservative law of solid-fluid mixture hydrodynamics as an ongoing effort to reduce siltation due to interstitial clogging in sand-filled reservoirs so as to increase water storage potential (Olufayo et al., 2011).

3 Conclusion

The domestic wastewater treatment process at the WWTP plays an important role as a wastewater treatment unit from densely populated areas in Yogyakarta. Domestic wastewater mostly consists of suspended solids, organic matter, pathogens, and nutrients. Inadequate treatment of domestic wastewater effluent can cause various diseases and jeopardize water quality. Challenges such as excessive sedimentation and eutrophication, if not controlled, can compromise the performance of the effluent treatment system. Various mathematical models have been used to study the interaction between wastewater components to solve various problems by applying techniques such as optimization, dynamic modeling, and control modeling.

This study is in line with SDG 6, which aims to improve effluent management efficiency to support clean water quality and safe sanitation, while reducing global environmental impacts. Mathematical modeling, especially dynamic models, plays an important role in understanding the relationship between waste particle components as well as how wastewater treatment systems work. This approach aims to observe changes and formulate effluent treatment strategies that are safe for health and the environment, while maintaining effluent quality for reuse. Numerical approaches, especially finite difference methods, are highly useful for overcoming the complexity of models that are difficult to solve exactly. This method excels due to its ease of application and geometric suitability to WWTPs. It allows for calculation results that closely match real values and can be simulated effectively. The results of this study provide opportunities for further study and resolution of various challenges related to WWTPs in various sectors. In the long term, the findings can serve as a foundation for the development of more effective and sustainable wastewater management policies.

Disclaimer (Artificial Intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

Highlights

- A review of mathematical models and their approach to WWTP challenges.
- Contamination, sedimentation and eutrophication are the main challenges in WWTPs.
- Optimization, control and dynamic models are the mathematical approaches often used in WWTPs.

Competing Interests

Authors have declared that no competing interests exist.

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